The Bivariate Double Rayleigh Distribution for Multichannel Time-Varying Systems

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***Abstract*—The bivariate double-Rayleigh distribution with non-identically distributed parameters is introduced and analyzed. Novel series expressions for the joint probability density, cumulative distribution, and the moments are derived. Based on this new distribution, a multichannel communication scenario is studied, where correlation exists between the actual channel and its corresponding estimate, under time varying channel assumption. In this context, the outage performance of the system under consideration is analytically evaluated, while for higher values of the average signal-to-noise ratio, closed-form results are derived. Finally, an analysis for total outage probability in varying-correlation conditions is also provided.**

***Index Terms*—Antenna selection, correlated (bivariate) double-Rayleigh distribution, outage probability, outdated CSI.**

I. Introduction

Double-Rayleigh (DR) distribution has been used for modeling double-scattering propagation conditions, which arise in cases where both the transmitter and the receiver are in motion, with their local scatterers be separated by a large distance [1]. This is why DR has been widely adopted in vehicle-to-vehicle (V2V) communications for modeling the non line-of-sight channel conditions. Based on this physically justified fading model, the performance of various communications systems has been analyzed, including transmit/receive diversity [2], [3] and cooperative/cognitive relaying [4]–[6]. For example, in [2], the channel capacity of a equal gain combiner in a V2V DR fading environment is studied. In [4], the performance of a cooperative diversity scheme with relay selection over cascaded Rayleigh fading has been investigated. However, a common assumption is all these works is that perfect channel state information (CSI) is available at the system and thus independent DR processes have been employed. In general, in contrast to other correlated fading models, e.g., Nakagami-m, Weibull [7], [8], the *correlated DR distribution* has not been thoroughly studied in the open technical literature.

In this letter, we present novel analytical expressions for the bivariate DR distribution, with non necessarily identical parameters and different correlation coefficients for the constitute distributions. In this context, new expressions for the joint probability density function (PDF), cumulative distribution function (CDF), and the moments are obtained. Capitalizing on the joint CDF, an application example on V2V communications has been presented. More specifically, the impact of outdated CSI on a multichannel system has been quantified. Similar investigations have been several times reported in the past for different scenarios, e.g., [9], [10]. However, previous works are characterized by cellular propagation conditions. Therefore, to the best of the authors' knowledge, the influence of outdated CSI on multichannel communication systems, operating in V2V environment, has not been investigated in the past and thus motivated this work.

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II. The Bivariate Double Rayleigh-Distribution

Let Z_1, Z_2 denote two DR envelopes with marginal PDF given by [11]

$$f_{Z_j}(x) = \frac{x}{\sigma_k^2 \sigma_\ell^2} K_0\left(\frac{x}{\sigma_k \sigma_\ell}\right) \tag{1}$$

where $j \in \{1,2\}$, $\sigma_k^2, \sigma_\ell^2$, with $k, \ell \in \{1,2,3,4\}$, are the scale parameters, and $K_0(\cdot)$ is the modified Bessel function of the second kind and 0th order [12, eq. (8.432/1)]. Since the DR envelope is defined as the product of two independent Rayleigh envelopes, i.e., X_i, with $i \in \{1,2,3,4\}$, Z_js are expressed as

$$Z_1 = X_1 \times X_2,\ Z_2 = X_3 \times X_4. \tag{2}$$

Next, X_1 and X_2 are considered to be correlated with X_3 and X_4, respectively, with joint PDF given by [7, eq. (6.2)]

$$\begin{aligned} f_{X_j, X_{j+2}}(x,y) = & \frac{xy}{\sigma_j^2 \sigma_{j+2}^2 (1-\rho_j^2)} \\ & \times \exp\left[-\frac{1/2}{(1-\rho_j^2)}\left(\frac{x^2}{\sigma_j^2} + \frac{y^2}{\sigma_{j+2}^2}\right)\right] I_0\left[\frac{\rho_j x y}{(1-\rho_j^2)\sigma_j \sigma_{j+2}}\right] \end{aligned} \tag{3}$$

where $I_v(\cdot)$ is the modified Bessel function of the first kind and order v [12, eq. (8.406/1)] and $(0 \leq \rho_j < 1)$ is the correlation coefficient of the underlying complex Gaussian random variables (RV)s. Based on (2), the joint PDF of Z_1 and Z_2 is given by

$$f_{Z_1,Z_2}(z,w) = \int_0^\infty \int_0^\infty \frac{f_{X_1,X_3}(x,y)}{xy} f_{X_2,X_4}\left(\frac{z}{x}, \frac{w}{y}\right) dx dy. \tag{4}$$

Substituting (3) in (4), employing the infinite series representation of the Bessel functions, [12, eq. (8.445)], making a change of variables and using [12, eq. (3.471/9)], yields the following expressions for the joint PDF of Z_1 and Z_2

$$\begin{aligned} f_{Z_1,Z_2}(z,w) = & \frac{1}{\hat{\rho}_{12} \hat{\sigma}_{1\text{-}4}^2} \sum_{i,j=0}^{\infty} \frac{\rho_1^{2i} \rho_2^{2j}}{(i!)^2 (j!)^2} \left(\frac{zw/4}{\hat{\rho}_{12} \hat{\sigma}_{1\text{-}4}}\right)^{j+i} \\ & \times zw K_{i-j}\left(\sqrt{\frac{1}{\hat{\rho}_{12}}} \frac{z}{\hat{\sigma}_{1\text{-}2}}\right) K_{i-j}\left(\sqrt{\frac{1}{\hat{\rho}_{12}}} \frac{w}{\hat{\sigma}_{3\text{-}4}}\right) \end{aligned} \tag{5}$$

where $\hat{\rho}_{12} = (1-\rho_1^2)(1-\rho_2^2)$, $\hat{\sigma}_{i\text{-}j} = \sigma_i \sigma_{i+1} \cdots \sigma_{j-1} \sigma_j$.

The joint CDF of Z_1 and Z_2 is defined as $F_{Z_1,Z_2}(x,y) \triangleq \int_0^x \int_0^y f_{Z_1,Z_2}(z,w)dzdw$. Substituting (5) in this definition, using the Meijer G-function representation for the Bessel function, i.e., [13, eq. (14)], and using [13, eq. (26)], the following generic expression for the joint CDF is obtained

$$F_{Z_1,Z_2}(x,y) = \frac{1}{\hat{\sigma}_{1\text{-}4}} \sum_{i,j=0}^{\infty} \frac{\rho_1^{2i}\rho_2^{2j}}{4(i!)^2(j!)^2} \left(\frac{xy/4}{\hat{\rho}_{12}\hat{\sigma}_{1\text{-}4}}\right)^{j+i+1} xy$$
$$\times \mathcal{G}_{1,3}^{2,1}\left(\frac{x^2/4}{\hat{\rho}_{12}\hat{\sigma}_{1\text{-}2}^2}\Big|_{\beta_{i,j},-\beta_{i,j},-1-\alpha_{i,j}}^{-\alpha_{i,j}}\right)\mathcal{G}_{1,3}^{2,1}\left(\frac{y^2/4}{\hat{\rho}_{12}\hat{\sigma}_{3\text{-}4}^2}\Big|_{\beta_{i,j},-\beta_{i,j},-1-\alpha_{i,j}}^{-\alpha_{i,j}}\right) \tag{6}$$

where $\alpha_{i,j} = \frac{i+j}{2}, \beta_{i,j} = \frac{i-j}{2}$, and $\mathcal{G}_{p,q}^{m,n}[\cdot|\cdot]$ denotes the Meijer's G-function [12, eq. (9.301)]. It is noted that Meijer G-functions are built-in function in many mathematical software packages, e.g., Mathematica, Maple, and thus can be easily evaluated. Moreover, substituting (5) in the definition of the joint moments $\mu_{Z_1,Z_2}(n_1,n_2) \triangleq \mathbb{E}\langle Z_1^{n_1} Z_2^{n_2}\rangle$, with $\mathbb{E}\langle\cdot\rangle$ denoting expectation, and using [12, eq. (6.561/16)], yields

$$\mu_{Z_1,Z_2}(n_1,n_2) = 2^{n_1+n_2}\hat{\rho}_{12}^{\frac{n_1+n_2}{2}+1}\hat{\sigma}_{1\text{-}2}^{n_1}\hat{\sigma}_{3\text{-}4}^{n_2}$$
$$\times \sum_{i,j=0}^{\infty}\frac{\rho_1^{2i}\rho_2^{2j}}{(i!j!)^2}\prod_{h=1}^{2}\Gamma\left(\frac{n_h}{2}+i+1\right)\Gamma\left(\frac{n_h}{2}+j+1\right) \tag{7}$$

where $\Gamma(\cdot)$ denotes the Gamma function [12, eq. (8.310/1)]. In addition, using the definition of the Gauss hypergeometric function [12, eq. (9.100)] and after some mathematical manipulation, the following closed-form expression for (7) is extracted

$$\mu_{Z_1,Z_2}(n_1,n_2) = 2^{n_1+n_2}\hat{\rho}_{12}^{\frac{n_1+n_2}{2}+1}\hat{\sigma}_{1\text{-}2}^{n_1}\hat{\sigma}_{3\text{-}4}^{n_2}$$
$$\times \prod_{h=1}^{2}\Gamma\left(\frac{n_h}{2}+1\right)^2 {}_2F_1\left(\frac{n_1}{2}+1,\frac{n_2}{2}+1,1,\rho_h^2\right) \tag{8}$$

with ${}_2F_1(\cdot)$ denoting the Gauss hypergeometric function. The correlation coefficient between Z_1 and Z_2 is defined as $\rho \triangleq \operatorname{cov}(Z_1,Z_2)/\sqrt{\operatorname{var}(Z_1)\operatorname{var}(Z_2)}$, where $\operatorname{cov}(\cdot)$ and $\operatorname{var}(\cdot)$ is the covariance and variance, respectively. Using (8) and [1, eq. (48)], ρ can be expressed as

$$\rho = \left[\hat{\rho}_{12}^2\prod_{i=1}^{2}{}_2F_1\left(\frac{3}{2},\frac{3}{2},1,\rho_i^2\right)-1\right]\frac{\pi^2/16}{1-\pi^2/16}. \tag{9}$$

It is noted that to the best of authors' knowledge all the previous derived analytical results have never been reported in the past. Moreover, the infinite series expressions presented above, converge for all values with practical interest, for the system's and channel's model parameters, using relatively small number of terms and in less time as compared to alternative techniques, e.g., by employing numerical integration, as it will depicted in Section IV. In addition, they can be also used for further analytical purposes, as it will be shown in the next section.

III. Multichannel System Outage Probability

In general, the analytical results derived in Section II can be used in various research fields where bivariate DR statistic is necessary. Here, the research focuses on a multichannel system operating in a V2V communication environment (modeled by the DR distribution) in the presence of additive white Gaussian noise (AWGN). In the scheme under consideration, an antenna selection mechanism has been adopted, which can be implemented at either the transmitter or the receiver side [14]. In both cases, the system selects the antenna that provides the highest instantaneous channel gain, a decision that is based on the estimation of the CSI. In this context, the channel gain Z_j that is available at the selection instance, due to the fast time varying nature of the medium, is different from the actual channel gain, $\hat{Z}_j$, at the transmission/reception instance [15]. The discrepancy between the channel gains is measured by the correlation coefficient ρ, defined in (9), and thus it depends on ρ_1 and ρ_2. In addition, ρ_1,ρ_2 depend on the maximum Doppler frequency f_{D_j} and the time delay due to the CSI feedback T_{D_j}.

In this case, the instantaneous signal-to-noise (SNR) at the jth diversity branch, with $j=1,2$, is defined as $\gamma_j = |Z_j|^2\frac{E_s}{N_0}$ with E_s denoting the transmitted symbol energy and N_0 noise variance. The corresponding average SNR is given by $\overline{\gamma}_j = \mathbb{E}\left\langle|Z_j|^2\right\rangle\frac{E_s}{N_0}$. Assuming independent and identically distributed (i.i.d.) fading conditions, i.e., $\overline{\gamma}_j=\overline{\gamma}$, the CDF of the actual SNR of the *selected branch* at the data transmission instance is expressed as

$$F_{\gamma_{\text{out}}}(\gamma) = \frac{2}{\overline{\gamma}}\sum_{i,j=0}^{\infty}\frac{\rho_1^{2i}\rho_2^{2j}}{(i!)^2(t!)^2}\left(\frac{1}{\hat{\rho}_{12}\overline{\gamma}}\right)^{\frac{j+i}{2}}\gamma^{\frac{i+j+2}{2}}\Bigg[i!j!$$
$$-\hat{\rho}_{12}^{1/2}\mathcal{G}_{2,2}^{2,2}\left(\hat{\rho}_{12}\Big|_{\frac{1}{2},-\frac{1}{2}}^{-i-\frac{1}{2},-j-\frac{1}{2}}\right)\Bigg]\mathcal{G}_{1,3}^{2,1}\left(\frac{\gamma}{\hat{\rho}_{12}\overline{\gamma}}\Big|_{\beta_{i,j},-\beta_{i,j},-1-\alpha_{i,j}}^{-\alpha_{i,j}}\right). \tag{10}$$

The proof for (10) is given in the Appendix. Based on the analytical expression for the CDF of the output SNR in (10), the performance of the system under consideration can be evaluated using the criterion of outage probability (OP) that is given by $P_{\text{out}} = F_{\gamma_{\text{out}}}(\gamma_{\text{th}})$.

A. Asymptotic Analysis

The exact results presented previously do not provide a clear physical insight of the system's performance. As such, the main concern is to derive an asymptotic closed-form expression for $F_{\gamma_{\text{out}}}(\gamma)$. Therefore, here, we focus on the high SNR regime to quantify the amount of performance variations, which are due to the correlation as well as to the receiver's architecture. In this context, assuming i.i.d. conditions, higher values of $\overline{\gamma}$ and using [16, eq. (03.02.06.0004.02)], the Bessel function in (3) can be approximated as $I_v(z) \approx \frac{1}{\Gamma(v+1)}\left(\frac{z}{2}\right)^v$. Using this approximated expression and the approach presented in Appendix, the following closed-form asymptotic expression for the CDF of γ_{out} is obtained

$$F_{\gamma_{\text{out}}}(\gamma) \approx \frac{4\sqrt{\hat{\rho}_{12}^{1/2}}}{\sqrt{\overline{\gamma}}}\left[\frac{\sqrt{\hat{\rho}_{12}\overline{\gamma}}}{2}-\sqrt{\gamma}K_1\left(\frac{2\sqrt{\gamma}}{\sqrt{\hat{\rho}_{12}\overline{\gamma}}}\right)\right]$$
$$\times\left[1-\frac{1}{\hat{\rho}_{12}}\Phi\left(1-\frac{1}{\hat{\rho}_{12}},1,2\right)\right] \tag{11}$$

where $\Phi(.,.,.)$ denotes the Lerch function [12, eq. (9.55)]. Moreover, assuming differential binary phase shift keying, the average bit error probability can be evaluated as $P_b =$

TABLE I
MINIMUM NUMBER OF TERMS OF (10) REQUIRED FOR OBTAINING ACCURACY BETTER THAN $\pm 10^{-5}$.

	$\overline{\gamma}=10dB$		$\overline{\gamma}=20dB$	
$\gamma(dB)$	$\rho_j=0.1$	$\rho_j=0.7$	$\rho_j=0.2$	$\rho_j=0.7$
0	2	11	1	8
5	2	13	1	10
10	2	15	2	12

$(1/2)\int_0^\infty \exp(-\gamma)F_{\gamma_{\rm out}}(\gamma)d\gamma$. Substituting, (11) in this integral and employing [12, eq. (6.631/3)], the following approximating expression for the ABEP is obtained

$$P_b \approx \hat{\rho}_{12}^{3/4}\left[1-\exp\left(\frac{1/2}{\hat{\rho}_{12}\overline{\gamma}}\right)W_{-1,1/2}\left(\frac{1}{\hat{\rho}_{12}\overline{\gamma}}\right)\right] \times\left[1-\frac{1}{\hat{\rho}_{12}}\Phi\left(1-\frac{1}{\hat{\rho}_{12}},1,2\right)\right] \tag{12}$$

where $W_{\mu,\nu}(\cdot)$ is the Whittaker function [12, eq. (9.220/4)].

B. Varying Correlation for Realistic Modeling

The previous analysis was based on deterministic values of ρ_j. However, in real-world situations, the vehicles (relative) velocity, i.e., $v_j = {\rm f}_{D_j}\lambda$, and/or the time of arrival between data transmissions, i.e., T_{D_j}, may continuously change in a random manner. Under these circumstances, ρ_js will also randomly vary. Therefore, it is reasonable to assume that correlation coefficients ρ_js are given by $\rho_j = g\left({\rm f}_{D_j}, T_{D_j}\right)$, e.g., assuming the classic Jakes spectrum $\rho_j = J_0\left(2\pi {\rm f}_{D_j}T_{D_j}\right)$, where $J_v(\cdot)$ is the Bessel function of the first kind [12, eq. (8.402)]. More specifically, we consider the case where T_{D_j}s are deterministic and thus the variations of ρ_js depend only on ${\rm f}_{D_j}$. The following analysis holds also in case where T_{D_j}s are stochastic and ${\rm f}_{D_j}$s deterministic. Under these assumptions, (10) expresses the conditional CDF $F_{\gamma_{\rm out}}\left(\gamma|\rho_1=r_1,\rho_2=r_2\right)$. In this context, the total OP involves the solution of the following integral:

$$F_{\gamma_{\rm out_{tot}}}(\gamma)=\iint_D F_{\gamma_{\rm out}}\left(\gamma|\rho_1=r_1,\rho_2=r_2\right) \times f_{\rho_1}(r_1)\,f_{\rho_2}(r_2)\,dr_1dr_2. \tag{13}$$

A generic analytical calculation of (13) is unfeasible. An alternative approach is to approximate (13), with proper change of variables from ρ_j to ${\rm f}_{D_j}$ and discretization of the Doppler domain into M equally spaced intervals resulting to the following formula

$$F_{\gamma_{\rm out_{tot}}}(\gamma)\simeq\frac{1}{A}\sum_{m=1}^{M}\sum_{n=1}^{M} f_{{\rm f}_{D_1}}\left({\rm f}_{D_1}(m)\right)f_{{\rm f}_{D_2}}\left({\rm f}_{D_2}(n)\right) \times F_{\gamma_{\rm out}}\left(\gamma|g\left({\rm f}_{D_1}(m)\right),g\left({\rm f}_{D_2}(n)\right)\right) \tag{14}$$

where ${\rm f}_{D_j}(m)$ is an M-point uniform sampling of the Doppler spread field, $f_{{\rm f}_{D_j}}(x)$ is the PDF of ${\rm f}_{D_j}$, and $A=\prod_{j=1}^{2}\sum_{m=1}^{M} f_{{\rm f}_{D_j}}\left({\rm f}_{D_j}(m)\right)$.

IV. Numerical Results

Firstly, the rate of convergence of the infinite series given in (10) has been investigated. More specifically, the minimum number of terms, which guarantees accuracy better than $\pm 10^{-5}$ is presented in Table I for different values of $\gamma,\overline{\gamma}$, and $\rho_j=\rho_1=\rho_2$. From this table it is clear that a relatively small number of terms is necessary to achieve an excellent accuracy. Moreover, this number of terms increases as γ and/or ρ_j increase as well as with the decrease of $\overline{\gamma}$. In Fig. 1, assuming $\overline{\gamma}=15$dB and $\gamma_{\rm th}=3$dB, a contour plot of the OP is depicted as a function of the two temporal correlation coefficients ρ_1 and ρ_2. It is noted that the antennas are *spatially uncorrelated*. In this plot, it is shown that as the correlation coefficients increase, e.g., feedback delay diminishes in a transmit antenna selection scenario, the OP improves with an increased rate, while the impact of ρ_1 and ρ_2 to the system's performance is identical. In the same figure, it is also depicted the probability of correct decision, which is plotted as a function of ρ_js and obtained via simulations. From this subfigure, it is evident that for lower values of ρ_j, the antenna selection becomes random, while clear diversity gain is expected only for $\rho_j > 0.8$. In Fig. 2, assuming $\gamma_{\rm th}=5$dB, the OP is plotted as a function of the average input SNR for different values of ρ_1,ρ_2. It is also shown that the performance improves as ρ_j increase. Moreover, in the same figure, the close agreement between the exact and the asymptotic (high SNR) OP is also depicted. It is noted that the approximation improves for lower values of

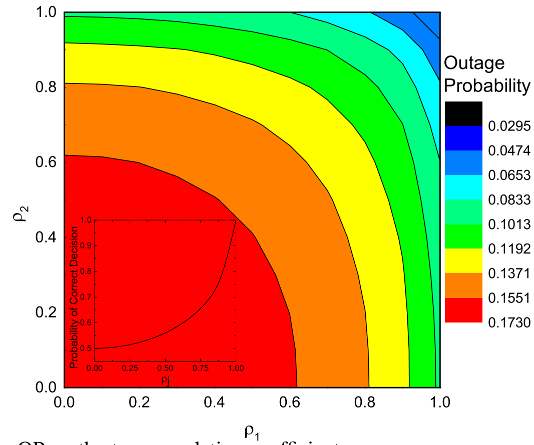

Fig. 1. OP vs the two correlation coefficients.

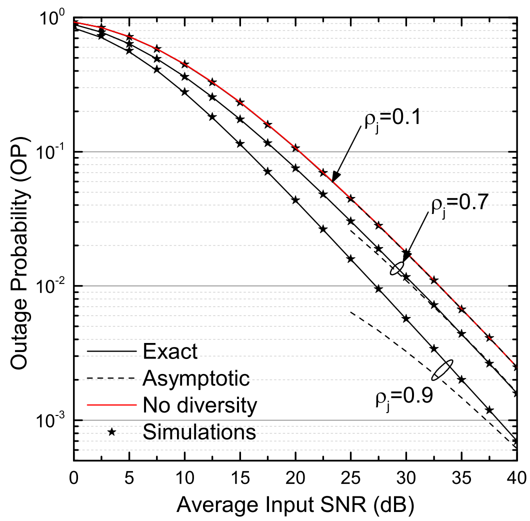

Fig. 2. OP vs the average input SNR.

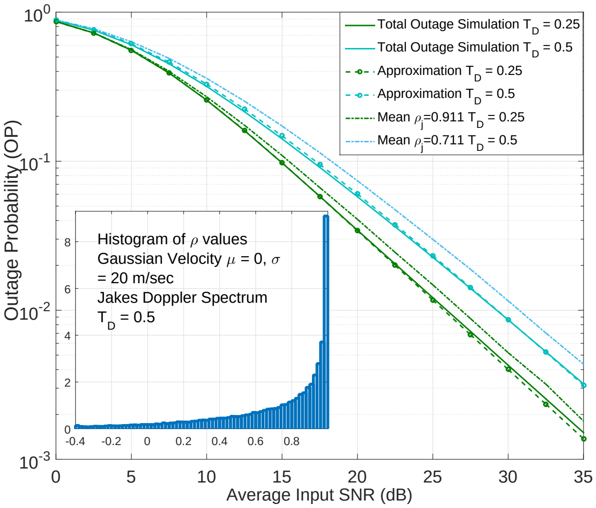

Fig. 3. OP vs the average input SNR for varying correlation.

ρ_js. In addition, for comparison purposes, the corresponding performance of a single (without diversity) receiver is also depicted. It is shown that for lower values of ρ_js, i.e., fast time varying channel, the diversity gain is lost. In this figure, computer simulations performance results are also included, verifying in all cases the validity of the proposed theoretical approach. In Fig. 3, assuming that the velocity of both ends of the link is a zero-mean Gaussian RV with $\sigma = 20$ m/sec (72 km/h), the total OP is calculated through simulation. It is noted that the distribution of ρ_js for the specific case is presented as a histogram in the subfigure. In addition, the approximation of (14) as well as the OP from (10), where the mean values of ρ_js are used, are also presented. It is interesting to note that (10) represents an upper bound of the total OP shown in (13), while the approximation given in (14) is quite close to (13).

V. Conclusions

Despite the wide adoption of the DR distribution for modeling V2V communication scenarios, correlated statistics have never been reported for this model. In this paper, the bivariate DR distribution with arbitrary correlations and non identical parameters was introduced and used to study the impact of outdated CSI on the OP performance of a V2V multichannel communication system. It was shown that the performance improves with the increase of the correlation between the channel gains at the selection and the data reception instances.

Appendix

In this Appendix, a proof for the derivation of (10) is presented. The CDF of the actual received SNR of the selected branch at the data transmission instance is expressed as

$$F_{\gamma_{\rm out}}(\gamma) = \int_0^\gamma \int_0^\infty \frac{f_{\gamma_j,\hat{\gamma}_j}(y,x)}{f_{\hat{\gamma}_j}(x)} f_{\hat{\gamma}_{\rm sd}}(x) dx dy \tag{A-1}$$

with $f_{\hat{\gamma}_{\rm sd}}(x) = 2 f_{\hat{\gamma}_j}(x) F_{\hat{\gamma}_{\rm j}}(x)$, $f_{\hat{\gamma}_j}(x) = 2/\overline{\gamma} K_0\left(2\sqrt{x/\overline{\gamma}}\right)$, and $F_{\hat{\gamma}_{\rm j}}(x) = 1 - 2x^{1/2}/\sqrt{\overline{\gamma}} K_1\left(2\sqrt{x/\overline{\gamma}}\right)$. Substituting (5) and $F_{\hat{\gamma}_{\rm j}}(x)$ in (A-1), the following integrals appear

$$\begin{aligned}
\mathcal{I}_1 &= \int_0^\gamma y^{\frac{j+i}{2}} K_{i-j} \sqrt{4y/\left(\hat{\rho}_{12}\overline{\gamma}\right)} dy \\
&\overset{(1)}{=} \frac{\gamma^{\frac{j+i}{2}+1}}{2} \mathcal{G}^{2,1}_{1,3}\left(\frac{\gamma}{\hat{\rho}_{12}\overline{\gamma}} \middle| \begin{matrix} -\alpha_{i,j} \\ \beta_{i,j}, -\beta_{i,j}, -1-\alpha_{i,j} \end{matrix}\right) \\
\mathcal{I}_2 &= \int_0^\infty x^{\frac{j+i}{2}} K_{i-j}\left(\sqrt{\frac{4x}{\hat{\rho}_{12}\overline{\gamma}}}\right) dx \overset{(2)}{=} \frac{i!j!/2}{\left(\hat{\rho}_{12}\overline{\gamma}\right)^{-\frac{i+j+2}{2}}} \\
\mathcal{I}_3 &= \int_0^\infty x^{\frac{j+i+1}{2}} K_{i-j}\left(2\sqrt{\frac{x}{\hat{\rho}_{12}\overline{\gamma}}}\right) K_1\left(2\sqrt{\frac{x}{\overline{\gamma}}}\right) dx \\
&\overset{(3)}{=} \frac{\left(\hat{\rho}_{12}\overline{\gamma}\right)^{\frac{j+i+3}{2}}}{4} \mathcal{G}^{2,2}_{2,2}\left(\hat{\rho}_{12} \middle| \begin{matrix} -i-\frac{1}{2}, -j-\frac{1}{2} \\ \frac{1}{2}, -\frac{1}{2} \end{matrix}\right).
\end{aligned} \tag{A-2}$$

In (A-2), for evaluating both $\mathcal{I}_1$ as well as $\mathcal{I}_3$, the Meijer G-function representation of the Bessel functions is employed, i.e., [13, eq. (14)]. Based on this representation, and with the aid of [13, eq. (26)] and [13, eq. (21)], the solutions of $\mathcal{I}_1$ and $\mathcal{I}_3$ are given in (1) and (3) of (A-2), respectively. Moreover, (2) holds due to [16, eq. (03.04.21.0116.01)]. Therefore, based on all these expressions, (10) is finally derived.

References

[1] J. Salo, H. M. El-Sallabi, and P. Vainikainen, “Statistical analysis of the multiple scattering radio channel,” *IEEE Trans. Antennas Propag.*, vol. 54, no. 11, pp. 3114–3124, Nov. 2006.

[2] B. Talha and M. Patzold, “On the statistical analysis of the channel capacity of double Rayleigh channels with equal gain combining in V2V communication systems,” in *IEEE Vehicular Technology Conference*, May 2010.

[3] A. Chelli, R. Hamdi, and M.-S. Alouini, “Channel modelling and performance analysis of V2I communication systems in blind bend scattering environments,” *Progress In Electromagnetics Research B*, vol. 57, pp. 233–251, 2014.

[4] M. Seyfi, S. Muhaidat, J. Liang, and M. Uysal, “Relay selection in dual-hop vehicular networks,” *IEEE Signal Process. Lett.*, vol. 18, no. 2, pp. 134–137, Feb. 2011.

[5] H. Ilhan and A. I. Akin, “Performance analysis of AF relaying cooperative systems with relay selection over double Rayleigh fading channels,” in *International Conference on Signal Processing and Communication Systems*, Dec. 2012.

[6] T. T. Duy, G. C. Alexandropoulos, V. T. Tung, V. N. Son, and T. Q. Duong, “Outage performance of cognitive cooperative networks with relay selection over double-Rayleigh fading channels,” *IET Communications*, vol. 10, no. 1, pp. 57–64, 2016.

[7] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New York: Wiley, 2005.

[8] N. C. Sagias and G. K. Karagiannidis, “Gaussian class multivariate Weibull distributions: Theory and applications in fading channels,” *IEEE Trans. Inf. Theory*, vol. 51, no. 10, pp. 3608–3619, Oct. 2005.

[9] L. Fan, X. Lei, R. Hu, and W. Seah, “Outdated relay selection in two-way relay network,” *IEEE Trans. Veh. Technol.*, vol. 62, no. 8, pp. 4051–4057, Oct. 2013.

[10] V. N. Q. Bao, T. Duong, and C. Tellambura, “On the performance of cognitive underlay multihop networks with imperfect channel state information,” *IEEE Trans. Wireless Commun.*, vol. 61, no. 12, pp. 4864–4873, Dec. 2013.

[11] I. Z. Kovács, P. C. Eggers, K. Olesen, and L. G. Petersen, “Investigations of outdoor-to-indoor mobile-to-mobile radio communication channels,” in *IEEE Vehicular Technology Conference*, 2002, pp. 430–434.

[12] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. New York: Academic Press, 2000.

[13] V. S. Adamchik and O. I. Marichev, “The algorithm for calculating integrals of hypergeometric type functions and its realization in REDUCE system,” in *International Conference on Symbolic and Algebraic Computation*, Tokyo, Japan, 1990, pp. 212–224.

[14] T. Gucluoglu and T. Duman, “Performance analysis of transmit and receive antenna selection over flat fading channels,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 8, pp. 3056–3065, Aug. 2008.

[15] V. Kristem, N. B. Mehta, and A. F. Molisch, “Optimal receive antenna selection in time-varying fading channels with practical training constraints,” *IEEE Trans. Commun.*, vol. 58, no. 7, pp. 2023–2034, Jul. 2010.

[16] The Wolfram Functions Site, 2016. [Online]. Available: http://functions.wolfram.com